



AMAZINGMATHS

# MATHEMAGIC

## - ABRACADABRA -

### How to do the Magic Trick

**Goal:**

Find the spectator's card.

**Preparation:**

The magician selects 21 cards from a deck of cards.

**Trick:**

1. The spectator chooses a card among the 21 cards and puts it back anywhere in the pile, without showing it to the magician.
2. The magician places the cards on the table from left to right into three columns and asks the spectator to tell him which column his card is in.
3. The magician takes the cards back.
4. The magician repeats steps 2 and 3 twice.
5. The magician spells the word "Abracadabra". For each letter, he places the card on top of the pile onto the table.
6. Once the word is spelled, the magician reveals to the spectator the last card that was placed on the table: it is the spectator's card!

**Materials:**

- Video of the trick
- 1 deck of cards



# MATHEMATICAL EXPLANATION

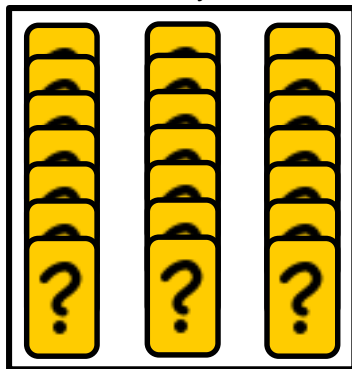


## Why this trick works.

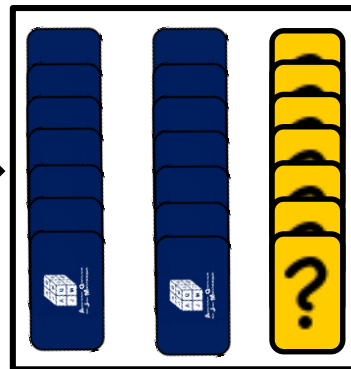
First, let's establish a code to differentiate the cards that can be the spectator's card from the ones that cannot:



Initially, because the spectator places his card back wherever he wants among the 20 other cards, the magician has no idea what kind it is; all the cards can be the spectator's card. But, as soon as the spectator tells the magician which column his card is in, the possibilities decrease from 21 cards to 7 cards. For example, if the spectator's card had initially been in the third column, the possibilities would have been reduced in this way:

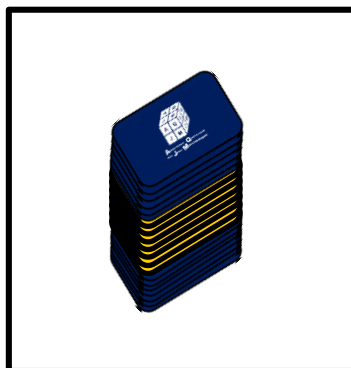


Picture 1

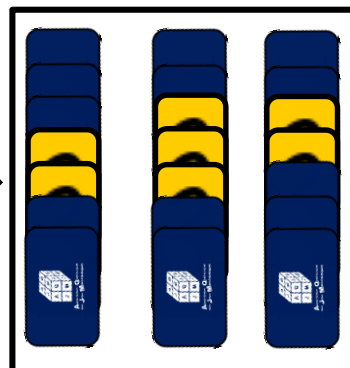


Picture 2

It is important to notice that when the magician takes the cards back, he always does it the same way: he takes the cards back one column at a time, always taking the column in which the spectator's card is 2<sup>nd</sup>. Then, he places the cards on the table from left to right to form three columns again. In the current example, the next arrangement would look like this:



Picture 3



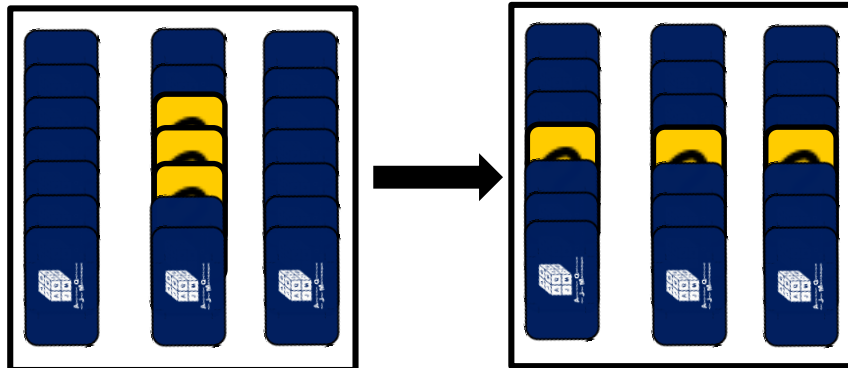
Picture 4

This way of doing aims at decreasing the number of cards that can be the spectator's in each column. Indeed, we notice that, in each column, there are two to three cards that can be the spectator's after the arrangement. Let's notice that these cards are placed either in the 3<sup>rd</sup>, 4<sup>th</sup> or 5<sup>th</sup> row in each column.

Let's suppose that the spectator's card is in the 2<sup>nd</sup> column. Then, the number of cards that can be the spectator's go from 7 to 3 (Picture 5). The magician then takes the cards back, making sure to take the column with the spectator's card 2<sup>nd</sup>, and he places the cards from left to right into 3 columns again (Picture 6).



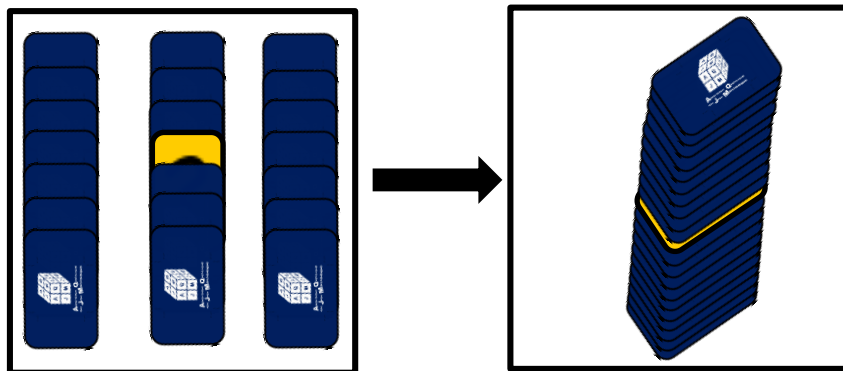
# MATHEMATICAL EXPLANATION



Picture 5

Picture 6

There is then only one card per column that can be the spectator's. Let's notice that each of those cards are in the 4<sup>th</sup> row of their column. When the spectator tells the magician which column his card is in, the magician knows which one it is since there is only one card left that can be the spectator's. Let's suppose the spectator's card is then in the 2<sup>nd</sup> column (*Picture 7*). For the trick to succeed, the spectator's card must be in the 11<sup>th</sup> position in the final pile, that is the row that corresponds to the number of letters in the word "Abracadabra". Since the spectator's card is in the 4<sup>th</sup> row of its column, the magician must place 7 cards on top of this column so the card finally ends up in the 11<sup>th</sup> row (*Picture 8*). To do so, the magician simply has to take back the column in which the spectator's card is second. He can then spell the word "Abracadabra", placing, for each letter, a card on the table. The last card to be placed is the spectator's card!



Picture 7

Picture 8