



MATHEMAGIC

- AMAZING ROLL OF THE DIE -



Educational Goals

- ❖ Develop logic
- ❖ Adopt a magic trick
- ❖ Highlight the playful potential of mathematics
- ❖ Formulate hypotheses and confirm them

Key Features of the Targeted Competencies

- ❖ To decode the elements that lend themselves to a mathematical treatment
- ❖ Form conjectures
- ❖ Elaborate a solution
- ❖ To form and apply a network of concepts and mathematical processes

Concepts Used

- ❖ Algebraic expressions
- ❖ Arithmetic (addition, subtraction, multiplication)
- ❖ Manipulation of algebraic expressions

Materials

- ❖ Video of the trick
- ❖ 3 small dice per team and ideally 3 big dice for the demonstration in front of the class
- ❖ Sheet of paper
- ❖ Pencils

Targeted Academic Level
Grades 9 to 11

Mathematical Field Concerned



Suggested Teaching Formula



Time Required
Approximately 40 minutes



SUGGESTED PROCESS



The goal of the activity is to let the students formulate a hypothesis about the magician's trick and try to confirm their hypothesis using a mathematical reasoning.

Step 1: Introduction (5 minutes)

Play the video of the magic trick once (www.amazingmaths.ulaval.ca).

Step 2: Recreate the magic trick and formulate hypotheses (15 minutes)

Refer to the Explanation Sheet "Amazing Roll of the Die" to do the magic trick for your students. Ask a student to be your spectator. Meanwhile, ask the other students to note the manipulations the magician is doing.

Afterwards, place the students in pairs so they can do the trick themselves: one plays the role of the magician and the other plays the spectator. They have to recreate the steps of the video until the magician's final reveal. (They do not know the magician's trick.)

Ask to try to formulate hypotheses on the link between the final number after all the operations done by the spectator and the value of each die.

Show several examples so they can see the link more easily. If they do not find it at all, suggest choosing the dice's values. It is a good idea to set two constant dice and have only one vary at a time. This allows to see the influence of a die on the final number. Here are some possibilities:

Value of the dice			Final number after the operations
die #1	die #2	die #3	
1	1	1	361
1	1	2	362
1	1	3	363
1	2	1	371
1	3	1	381
2	1	1	461
3	1	1	561

Make them understand that the order in which we dictate the dice to the magician is important, otherwise the final number is not the same. (For example, see the table above for the sequence 1, 2, 1 versus 2, 1, 1.)

The expected hypothesis is that, if we subtract 250 from the final number, we get the combination of the number formed by the value of the first die, followed by the second one and the last one.



SUGGESTED PROCESS



If the students do not find the magician's trick, the next step will help them.

Step 3: Confirming the hypothesis (15 minutes)

Now that the students' hypotheses were formulated, they must confirm them.

Because the numbers displayed on the dice are unknown and that the spectator does the arithmetic operations with these numbers, they can continue their reasoning algebraically.

Hint the students about the following points:

- Since the three values of the dice are unknown, we can associate a variable to each value. For example, we can write:
 $x :=$ value of the first die;
 $y :=$ value of the second die;
 $z :=$ value of the third die.
- Each arithmetic operation leads to a new algebraic expression that depends on one value, two values or all the values of the dice.
- After all the operations, we get an algebraic expression that depends on the three values ($100x + 250 + 10y + z$).
- What can we conclude from this algebraic expression?
Expected answer: When we remove 250 from the final number, we get

$$100x + 250 + 10y + z - 250 = 100x + 10y + z.$$

- Why can the magician deduce the dice's values? What are worth a hundred, a ten and a unit? Is there a link with the place value of the numbers?

Step 4: Reveal the solution (5 minutes)

Refer to the Explanation Sheet for the trick "Amazing Roll of the Die" for a detailed solution.