MATHEMAGIC

- ARITHMETIC TRIANGLE -

AMAZINGMATHS

Materials :

- Magic trick video
- 1 deck of playing cards
- Illustration of triangle

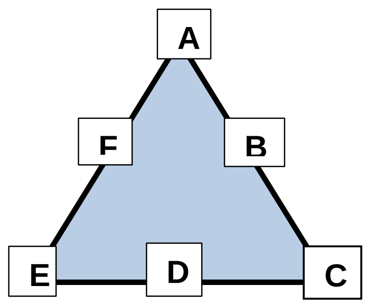
How to do the Magic Trick

Goal :

Find the value of the card selected by the spectator.
Note: The value of the Jack, the Queen, the King and the Ace are respectively 11, 12, 13 and 1.

Preparation :

- On a sheet of paper, recreate the following arithmetic triangle needed for this trick:



- Remove the Jokers from the deck of cards.

Trick :

1. The spectator selects four of the 52 cards and hides them from the magician.
2. Among the four cards, he chooses one card and keeps its value in mind. He puts the four cards aside, face down.
3. While the magician is turned around, the spectator spreads some cards on the letters of the triangle as follows:

    a) He places a number of cards equivalent to the value of the chosen card on each vertex of the triangle (the letters A, C, and E).

    b) He distributes the remaining cards one at a time by alternating one on B, one on D, and one on F, until no cards remain.

4. The magician asks the spectator to choose one of the three sides of the triangle, to count the number of cards on that side and to reveal this number.

5. The magician can then tell the value of the spectator's card.

    To do this, the magician simply has to subtract 16 from the number of cards appearing on the side of the triangle.

MATHEMATICAL EXPLANATION

Why this trick works

First possible solution (algebraic):

At the beginning of the trick, the spectator selects 4 cards and removes them from the deck. Thus, the trick takes place with 48 cards (52 - 4).

Let's set the following variable:

x := value of the card chosen by the spectator.

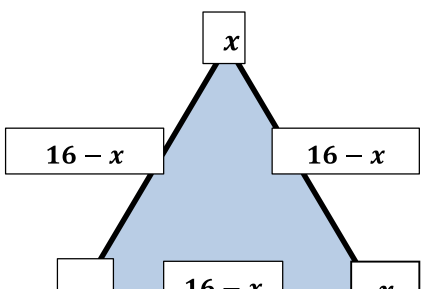
We know that after step 3. a), there will be exactly x cards on vertex A, x cards on vertex C, and x cards on vertex E.

For step 3. b), as the trick begins with 48 cards and the spectator removes x cards for the 3 vertices, the spectator distributes for the sides B, D, and F a total of:

$48 - 3x$ cards.

Moreover, as the spectator alternates the distribution of the remaining cards on the other three letters, we know that they will have the same number of cards. In other words, we must divide the number of cards to be distributed by 3 to know the number of cards that will be on the boxes B, D, F.

$$\frac{48-3x}{3} = \frac{48}{3} - \frac{3x}{3} = 16 - x.$$



Finally, each side of the triangle is made of two vertices and one of the letters B, D, or F.

Knowing what we just said earlier, if we ask:

n:= number of cards on one side of the triangle,

we obtain:

$$n = x + x + (16 - x) = 2x + 16 - x = x + 16$$

$$\Rightarrow \; n = x + 16$$
$$\Rightarrow \; x = 16 - n.$$

So, when the spectator reveals the number of cards on one side of the triangle (n), the magician must simply subtract 16 from this number to know the value of the card (x).

MATHEMATICAL EXPLANATION

Why this trick works

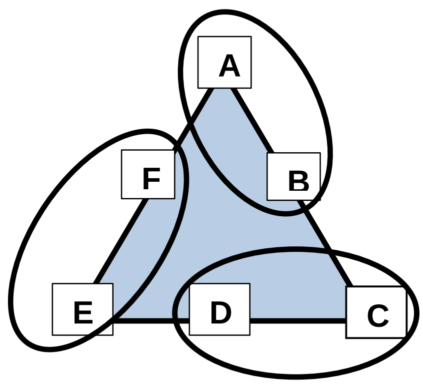
Second possible solution:

At the beginning of the trick, the spectator selects 4 cards and removes them from the deck. Thus, the trick takes place with 48 cards (52 - 4).

We know that the number of cards on the vertices (letter A, C, and E) is the same.

Also, the way of distributing the cards results from the fact that the number of cards on letters B, D, and F is the same.

We can therefore conclude that the 3 ovals of the figure below each have the same number of cards (each oval consists of 2 letters: a letter among A, C, and E and a letter among B, D, and F).



Similarly, as all the cards are distributed on the letters, we can conclude that each oval has one third of 48, which is 16 cards.

Now, each side is made of an oval and a letter of a vertex. Remembering that the number of cards on the letters at the vertex is equal to the value of the card chosen by the spectator, we know that <u>the number of cards on one side of the triangle is 16 (number of cards in the oval) added to the value of the card (letter at the vertex).</u>

Thus, we conclude that the value of the card is equal to the number of cards on one of the sides of the triangle, from which we subtract 16.