MATHEMAGIC

- ARITHMETIC TRIANGLE -

Educational Goals

- Develop logic
- Highlight the playful potential of mathematics
- Identify the information needed to solve a mathematical problem
- Use algebra to solve a mathematical problem

Key Features of the Targeted Competencies

- To decode the elements of the situational problem
- To model the situational problem
- To apply different strategies to work out a solution
- To validate the solution
- To define the elements of the mathematical situation
- To mobilize mathematical concepts and processes appropriate to the given situation

Concepts Used

- Algebraic expressions
- Arithmetic operations (addition, subtraction, multiplication)
- Manipulation of algebraic expressions

Materials

- Magic trick video
- 1 deck of playing cards
- Paper
- Pencils

Targeted Academic Level
Grades 9 to 11

Mathematical Field Concerned

Suggested Teaching Method

Time Required
Approximately 40 minutes

SUGGESTED PROCESS

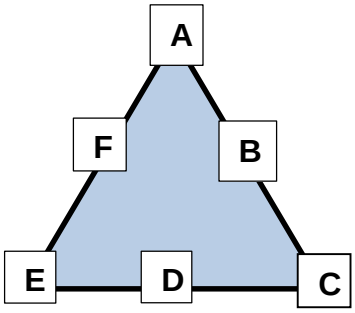
Step 1: Introduction (5 minutes)

Play the magic trick video once (www.amazingmaths.ulaval.ca).

Step 2: Recreate the magic trick (30 minutes)

Consult the ‘’Arithmetic Triangle’’ Explanation Sheet to perform the trick for your students. Ask a student to be your spectator.

Place the students in pairs so they can do the trick themselves: one plays the role of the magician and the other plays the role of the spectator. They can take a sheet of paper to reproduce the arithmetic triangle:



They must reproduce the steps of the video until the magician’s reveal. (They do not know what the magician’s trick is).

Once all the cards are placed on the arithmetic triangle, proceed to the step of searching for the solution. They can use the example in front of them to find out how the trick works.

Step 3: Reveal the solution (15 minutes)

The first step is to search for the information known by the magician. To guide the student’s thought process, ask the following questions:

- How many cards are spread on the triangle in total? (48 cards)
- Is there a link between the number of cards placed on certain letters? Do some letters have the same number of cards? If yes, why? (The vertices A, C, and E have the same number of cards, as do the boxes B, D, and F).
    - For vertices A, C, and E, the magician asks the spectator to place the same number of cards (the value sought).

SUGGESTED PROCESS

- For the boxes B, D, and F, the distribution of the cards results from the fact that there is the same number on each of these boxes. However, this is possible only because the rest of the cards for distribution on these boxes is a multiple of 3.

- Why is the number of cards to be placed on boxes B, D, and F a multiple of 3?
  *(The total number of cards to hand out is 48; this number is a multiple of 3. We then remove another multiple of 3 from it, since the same number of cards are placed on **3** vertices, the rest of the cards must therefore be a multiple of 3.)*

- Why does the magician ask the spectator to choose a side? Does this choice change anything? *(No, that does not change anything.)*

  To answer this question, they can use the final layout of the cards on the triangle that they created. They can confirm that it does not matter by counting the total number of cards on each side.

- Why is the total of cards the same for each side?
  (The number of cards on the letters B, D, and F is the same and the number of cards on the vertices A, C, and E is the same. One side always consists of 2 vertices (letters A, C, and E) and one of the letters B, D and F.)

The following 3 pieces of information are known by the magician:

1. There is a total of 48 cards to be distributed on the triangle.
2. The number of cards placed on the letters A, C, and E is the same.
3. The number of cards placed on the letters B, D, and F is the same.

The second step is to look for the magician's trick. The trick can be solved in several ways (see Explanation Sheet for two possible solutions). Let them think about the solution, knowing the information previously found.

If students do not know where to start, suggest marking the value of the chosen card as unknown and continue the reflection algebraically:

$x :=$ the value of the spectator's card.

You can help them by asking the following questions:

- Can we represent the number of cards on the letters A, C, and E according to x?
  (*Yes, it is x because the magician asks to place on these letters the number of cards corresponding to the value of the card sought (x).)*

- Can we also represent the number of cards on the letters B, D, and F according to x? *(Yes, it is $16-x$. For the explanation, see the Explanation Sheet.)*

SUGGESTED PROCESS

- Having passed the first two points, can we express the total number of cards on one side according to x? If so, which arithmetic operation should the magician do if he knows the total number of cards on one side?
  (Yes, it is $16 + x$ since each side consists of 2 vertices (letters A, C, and E) and one of the letters (B, D, or F).)

Step 4: Reveal the solution (5 minutes)

Refer to the ‘’The Arithmetic Triangle’’ Explanation Sheet to reveal the solution.