



# MathEMagic

*– Birds of a Feather Flock Together –*



## Educational Goals

- ❖ Develop logic
- ❖ Adopt a magic trick
- ❖ Observe the influence of the magician's manipulations on the positioning of the cards

## Key Features of the Targeted Competencies

- ❖ To decode the elements of the situational problem
- ❖ To apply different strategies to work out a solution
- ❖ To define the elements of the mathematical situation
- ❖ To mobilize mathematical concepts and processes appropriate to the given situation

## Concepts Used

- ❖ Positioning
- ❖ Cycles

## Materials

- ❖ Magic trick video
- ❖ 1 deck of playing cards or several sets of cards numbered from 1 to 5 (see [Appendix 1](#))

**Targeted Academic Level**  
Grades 7-11

**Mathematical Field Concerned**



**Suggested Teaching Methods**



**Time Required**  
25 to 40 minutes



# Suggested Process



## Before starting the activity:

Place students in teams of two or three. There must be a set of black cards and a set of red cards for each team. (See [Appendix 1](#))

## Step 1: Introduction (5 minutes)

Play the magic trick video once ([www.amazingmaths.ulaval.ca](http://www.amazingmaths.ulaval.ca)).

Note: While solving the trick with the students, you can use sequences of the trick's solution video to illustrate your explanations.

## Step 2: Recreate the trick in class - in small groups (10 minutes)

Ask the students to recreate the trick in small groups: one student takes on the role of the magician and the other the role of the spectator.

Point out to the students that this trick works regardless of the choices that have been made by the spectator (the pile that is placed on top, the number of cuts that have been made and the choices of the piles to make the changes of position).

## Step 3: Dissecting the steps of the trick – in small groups and the whole class together (20 minutes)

For each step of the trick:

- Tell the students which step is being observed;
- Ask them to analyze this step in small groups to determine its effect on the layout of the cards and on the trick;
- Review with the class:
  - Ask a few teams to tell their assumption(s).
  - Explain what there is to observe in the step studied.

Here are each of the steps to dissect:

- 1) The spectator chooses which pile goes on top.

**Effect on the cards:** No effect.

**Explanation:** The colour of the cards is not important. The goal of the trick is to assemble the similar cards' values together. No matter which pile is placed on top, the effect is the same: the order of the cards is preserved (**Ace – 2 – 3 – 4 – 5 – Ace – 2 – 3 – 4 – 5**). The values of the first 5 cards are identical to the values of the 5 last cards.





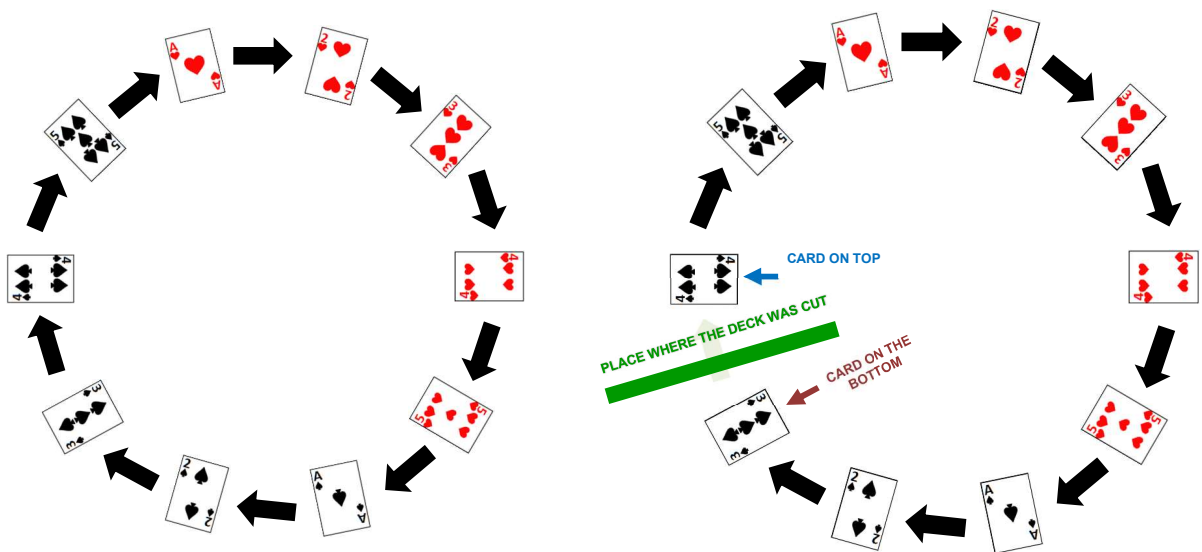
## Suggested Process (continued)



2) The spectator cuts the deck of cards as many times as he wants.

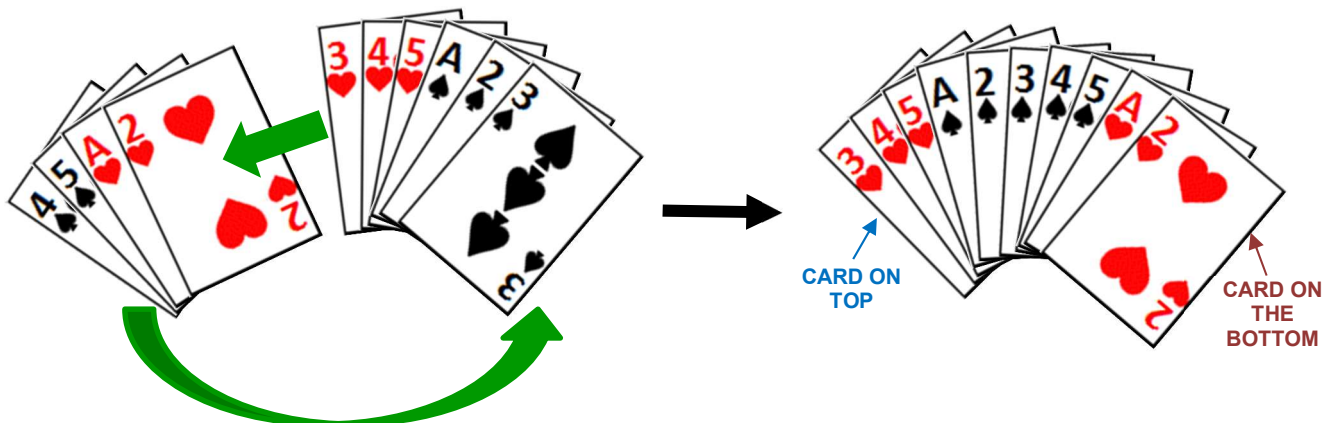
**Effect on the cards:** The card on top of the deck changes, but the order of the cards remains the same.

**Explanation:** The pile of cards can be represented as a cycle. When the spectator cuts the pile, he selects the number of cards of his choice and places them after the last card without changing their order. There is then a new card on top of the pile, but the order of the cards is the same as before the cut.



After this step, the value of the card on top can be different than the Ace but the following characteristics are always respected:

- The value of the first 5 cards are the same as those of the last 5 cards;
- Thus, there are always 4 cards in between 2 cards of the same value.





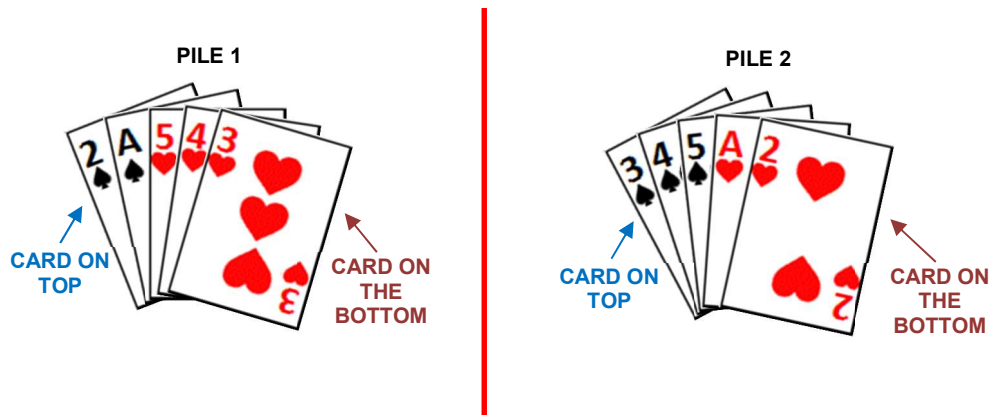
## Suggested Process (continued)



- 3) The magician separates the cards in 2 piles of 5 cards each.

**Effect on the cards:** The value of the cards are reflections of one another in each pile.

**Explanation:** The magician places the first 5 cards one by one for the first pile, which reverses the order of the cards completely. The card that was initially on the top is now at the end of the first pile. The last 5 cards are placed as they are on the table, without changing their order. So, the value of the card in first position in the first pile will be found in 5<sup>th</sup> position in the second pile, and so on.



- 4) The spectator does 4 changes of position in the pile of his choice. The magician removes 2 cards on top of each pile and pairs them.

**Effect on the cards:** The value of the cards on top of each pile is identical.

**Explanation:** Due to the fact that the cards in each pile mirror each other, the choice of the pile to make the changes of position is not important: these changes will inevitably place 2 cards of the same value on top of each pile. The magician removes the 2 cards and pairs them.



## Suggested Process (continued)



- 5) The spectator makes one change of position less than in the previous step and repeats this operation until there is only one card left in each pile. After each series of changes of position, the magician removes the cards on top of each of the piles and pairs them.

**Effect on the cards:** The effect is the same as that of the previous step.

**Explanation:** The spectator must make one change of position less than in the previous step, as there is one less card in each pile. But, the values of the cards always mirror each other from one pack to another. The changes of position will allow to place 3 cards of the same value on top of each pile and so on.

- 6) The magician reveals the pairs that were put together: all the cards of the same value are paired!



### TO GO FURTHER!

Would the trick still work if the spectator decided to distribute the changes of position in both piles? (For example, instead of making 4 changes of position in one pile, he makes 1 in the first pile and 3 in the second?).



# Appendix 1

