

# MATHEMAGIC

## - MULTIPLICATIVE PAIRS -



AMAZINGMATHS

### Materials:

- Magic trick video
- Cards in appendix
- 1 writing slate

## How to do the Magic Trick

### Goal:

Find the spectator's two cards.

### Preparation:

The magician only uses the cards in the appendix. For the trick, you have to print the appendix twice and cut out the cards.

### Trick:

1. The spectator draws 3 cards that he keeps hidden. He must choose two, making sure that the multiplication of the numbers on the two cards does not make 60. The goal of the magician will be to find the exact value of each of these two cards.
2. The magician reveals the cards he has one by one on the table, placing them side by side. If two of these cards have a product of 60, he puts down a new card, face up, on each of the two cards. If there are two possibilities of pairs of cards which product is 60, we must choose one to be covered with new cards. The choice does not matter.

The magician does this until all the cards are on the table.

CAREFUL: If there is only one card, it must be placed alone on the table next to the others.

3. When all the cards are placed on the table, we are interested in the cards that are on top of the piles. When two of them form a pair which product is 60, we eliminate their piles. At the end of this step, there should be 2 piles left on the table. The magician can then determine the value of each of the two cards drawn.

*(To do so, for each of the two remaining piles, he must make the division:  $60 \div$  value of the card at the top of the pile. The two results obtained correspond to the values of the two cards drawn by the spectator).*



# MATHEMATICAL EXPLANATION



## Why This Trick Works.

The product of the red cards in the pile can be written as follows:

$$2 \times 2 \times 3 \times 3 \times 4 \times 4 \times 5 \times 5 \times 6 \times 6 \times 10 \times 10 \times 12 \times 12 \times 15 \times 15 \times 20 \times 20 \times 30 \times 30.$$

It can be rewritten as 10 pairs which product is 60:

$$(2 \times 30) \times (2 \times 30) \times (3 \times 20) \times (3 \times 20) \times (4 \times 15) \times (4 \times 15) \times (5 \times 12) \times (5 \times 12) \times (6 \times 10) \times (6 \times 10).$$

Or even as:  $60 \times 60 \times 60.$

So, each card can be paired with another to get a product of 60.

By drawing two cards, we remove them from the total product of the cards in the pile. For example, if someone draws a 2 and a 15, the product is written as follows

$$30 \times (2 \times 30) \times (3 \times 20) \times (3 \times 20) \times 4 \times (4 \times 15) \times (5 \times 12) \times (5 \times 12) \times (6 \times 10) \times (6 \times 10).$$

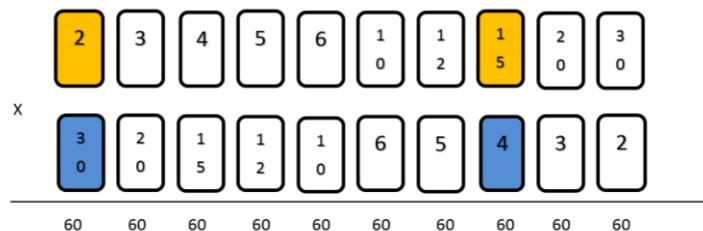
Which is equal to:  $30 \times 60 \times 60 \times 60 \times 4 \times 60 \times 60 \times 60 \times 60 \times 60.$

To find  $60 \times 60 \times 60$  we must multiply the single numbers by 2 and 15.

This allows us to find the value of the cards that were removed.

## Detailed Explanations

Regardless of the value of the chosen card, there is always a complementary card associated with it in the pile for the product of the cards to make 60.



We make pairs with two cards which product is 60. When the spectator chooses two cards among the three he has drawn, two pairs which product is 60 cannot be completed. By making pairs which product is 60, the magician finds each "complete" pair. As he removes all the "complete" pairs, there are two cards left at the end which product is not 60. These two cards are the ones that go with the cards chosen by the spectator.

This allows us to find the value of the drawn cards.

- The value of one card on the table multiplied by the value of one chosen card is 60.
- 60 divided by the value of the card on the table is the value of the chosen card.

So, we find the two cards.

$$\text{Ex.: } 60 \div 2 = 30 \text{ and } 60 \div 15 = 4.$$

The two cards drawn at the beginning are 30 and 4!



# APPENDIX



30

20

15

12

10

2

3

4

5

6