MATHEMAGIC
- QUICKER THAN THE CALCULATOR -

How to do the Magic Trick

Goal:

Calculate the product of the given numbers before the spectator can find it using the calculator.

Trick:

1. The magician gives a calculator to the spectator.

2. The spectator writes the three-digit number on the calculator.

3. The magician indicates the three operations that the spectator must perform on the calculator (x 7, x 11, x 13).

   Note: To prevent the spectator from making a mistake, the magician can ask him to write the number on the calculator before explaining the instructions so he does not forget it.

4. On the magician's signal, the spectator announces the chosen three-digit number and performs the operations on that number using the calculator.

5. Meanwhile, the magician writes the product on a writing slate. He indicates when he is done, trying to do the calculation faster than the spectator.

6. When the spectator has finished his calculation, the magician shows him the number written on the writing slate.

7. The number on the calculator and the number on the writing slate match!

MATHEMATICAL EXPLANATION

Why This Trick Works.

The magician is able to find the final product quickly because the operations requested are particular.

Note that the product of 7 x 11 x 13 is 1 001. So, the operations that the spectator performs are just another way to multiply his number by 1 001.

Let's concretely analyze what happens when a three-digit number is multiplied by 1 001. If the spectator had chosen the number 526, the product of his number by 7, by 11, and by 13 would have been 526 526. In fact, the product is composed of the spectator's numbe in the thousands position and the spectator's number in the units position (526 526 = 526 thousands + 526 units). Regardless of the number chosen by the spectator, this statement will be true. The final number will be composed of the spectator's number in the units position. We can convince ourselves by doing the following operation:

$$
\begin{array}{r}
1\ 001 \\
\times\ \ 526 \\
\hline
6\ 006 \\
+\ 20\ 020 \\
500\ 500 \\
\hline
526\ 526
\end{array}
$$

What would the result have been if the spectator had changed the order of the numbers multiplied? For example, if the spectator had chosen to do the operations in the following order:

7 x 526 x 13 x 11.

Would the result have been the same?

The answer is yes. Regardless of the order in which the numbers are multiplied, the product will always be formed from the spectator's number in the thousands position and the spectator's number in the units position. This result is explained by the **commutative property** of multiplication according to which the order of the terms in a multiplication does not matter.

To successfully write the product before the spectator can find it with the calculator, the magician must only write the spectator's number twice, once in the thousands position and once in the units position.