



MATHEMAGIC - SQUARED -



Educational Goals

- ❖ Develop logic
- ❖ Adopt a magic trick
- ❖ Highlight the playful potential of mathematics
- ❖ Formulate hypotheses and confirm them

Key Features of the Targeted Competencies

- ❖ To decode the elements of the situational problem
- ❖ To form conjectures
- ❖ To elaborate a solution
- ❖ To build and operate networks of mathematical concepts and processes

Concepts Used

- ❖ Algebraic expressions
- ❖ Arithmetic operations (addition, subtraction, multiplication)
- ❖ Manipulating algebraic expressions

Materials

- ❖ Magic trick video
- ❖ Paper
- ❖ Pencils

Targeted Academic Level
Grades 9 to 11

Mathematical Field Concerned



Suggested Teaching Method



Time Required
Approximately 40 minutes



SUGGESTED PROCESS



The purpose of this activity is to let the students formulate a hypothesis about the magician's trick and validate it with mathematical reasoning.

Step 1: Introduction (5 minutes)

Play the magic trick video once (www.amazingmaths.ulaval.ca).

Step 2: Recreate the magic trick and formulate hypotheses (15 minutes)

Refer to the "Squared" Explanation Sheet to perform the trick to your students. Ask a student to choose the first number. At each manipulation, choose a different student in order to have different students participate.

Then, place the students in pairs so that they can perform the trick themselves: one plays the role of the magician and the other plays the role of the spectator. They must recreate the steps performed in the video up to the magician's final reveal (they do not know what the magician's trick is).

Give them as an objective to formulate hypotheses about how the magician can find the starting number only from the result of subtracting the number B by the number A (*see the trick to understand what these numbers are*). To guide the reflection, you can ask the following questions:

- What information does the magician ask the spectator to find the starting number?
- What link can there be between this information and the starting number?
- Is there an arithmetic operation that seems to link the starting number and the subtraction of the number B by the number A?

Make several examples so that they can see the link more easily. If they do not find it at all, suggest trying the trick with consecutive numbers or multiples of 10. Here are some examples:

Number chosen by the spectator	Subtraction of number B by number A
12	240
13	260
14	280
23	460
24	480
25	500
10	200
20	400
30	600

The expected hypothesis is that the subtraction of the number B by the number A is 20 times greater than the number given by the magician.



SUGGESTED PROCESS



Step 3: Validate the hypothesis (15 minutes)

Now that the students' assumptions have been made, they have to confirm them.

Since the starting number is unknown and the spectator makes mathematical manipulations with it, they can continue their reflection algebraically.

Help the students with the following points:

- Since the starting number is unknown, we can associate a variable with it (let's assign $x :=$ the starting number).
- The number A depends on the starting number, meaning that we cannot formulate the number A without knowing the starting number. $(A(x) = (x + 5)^2)$
- The number B equally depends on the starting number $(B(x) = (x - 5)^2)$.
- By having formulated the link between A and the starting number as well as B and the starting number, the subtraction of the number B by the number A also depends on the starting number. $(B - A)(x) = (x + 5)^2 - (x - 5)^2$.
- Can we simplify the subtraction? What result do we get?
($B - A = 20x$.)

Step 4: Reveal the solution (5 minutes)

Refer to the "Squared" Explanation Sheet for a detailed solution.

Since you have shown that the subtraction of the number B by the number A is 20 times greater than the starting number for any arbitrary (variable) value, you have found the magician's trick!

To go further

Here are some interesting questions to stimulate the reflection:

- Should the number chosen by the spectator be between 1 and 50? Can it be as big as desired?
- Does the number that the spectator adds to his starting number to get A (5) must be the same number he subtracts from his starting number to get B (5 again)?
- What would the magician's trick have been if the number used to add and subtract was not 5, but rather 6 or 10?