MATHEMAGIC

-THE HAT TRICK-
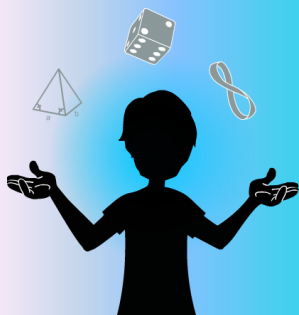
How to do the Magic Trick

Goal:

Find both of the spectator's cards.

Trick:

1. The magician randomly gives 11 cards to the spectator.

2. The magician asks the spectator to choose 3 cards among the 11 he gave him and to memorize them.

3. The spectator puts the 3 cards he chose back on top of the deck (over the 41 cards left).

4. The 8 cards that were not chosen are then put back on top of the deck (over the spectator's cards).

5. The magician asks the spectator to choose a number from 10 to 19. He then puts down the number of cards corresponding to the number chosen by the spectator, forming a pile. Let's name it **pile A**.

6. Next, the magican asks the spectator to calculate the sum of the two digits forming the number he chose. He takes **pile A** and puts back on the deck the number of cards corresponding to the sum previously calculated.

   *N.B. When the magician puts the cards of **pile A** back on the deck, he takes the cards one after the other starting from the top of **pile A**.*

   The next card in **pile A** is one of the 2 cards chosen by the spectator. The magician takes it and sets it aside, without revealing it. He places the cards left from **pile A** on top of the deck.

7. The magician repeats steps 5 to 7 two more times.

8. The magician then reveals the 3 cards set aside. They are the 3 cards chosen by the spectator.

Why this trick works.

The key to the explanation for this trick is the 9th position in the deck. To follow which card is in this position, it will always be indicated by a star. ⭐

When the spectator's cards are placed on the deck and the 8 cards that were not chosen are added, the spectator's three cards end up in the 9th, the 10th and the 11th position in the deck.

Spectator's cards	The 8 cards that were not chosen

| 11 | 10 | 9 ⭐ | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | |

**Bottom**   **Top**
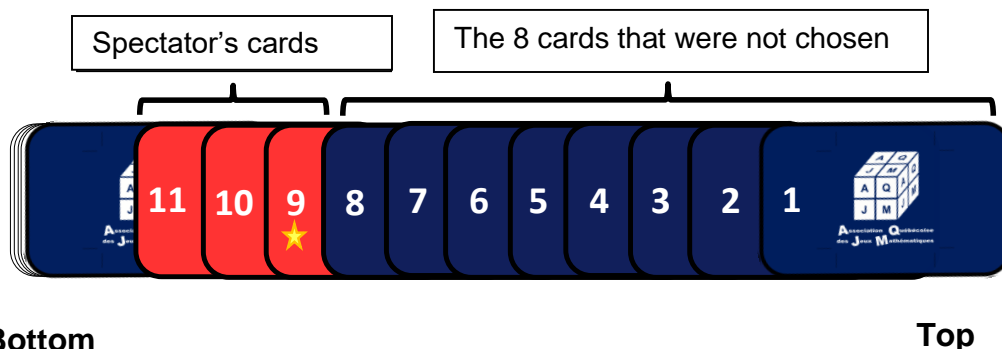
The magician's goal is to remove the card in the 9th position. When the spectator chooses a number from 10 to 19, the card in the 9th position then ends up in the pile set on the table (called **pile A** in the trick's description previously provided).

Let's suppose the spectator chooses the number 15. The magician places 15 cards into a pile on the table (called **pile A**). How many cards does he have to put back on the deck to reach the 9th position? He must put 6 cards back on the deck to reach the card in the 9th position.

$$15 - 9 = 6.$$

Let's notice that 15 is made of the digit 1 and the digit 5. The sum of these two digits is: $1 + 5 = 6$. This sum corresponds to the number of cards to put back on the deck. Refer to the section "**Why do all the numbers from 10 to 19 work?**" below to have more explanation on the matter.

Pile A

Spectator's cards

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 ⭐ | 10 | 11 | 12 | 13 | 14 | 15 | |

6 cards

15 cards
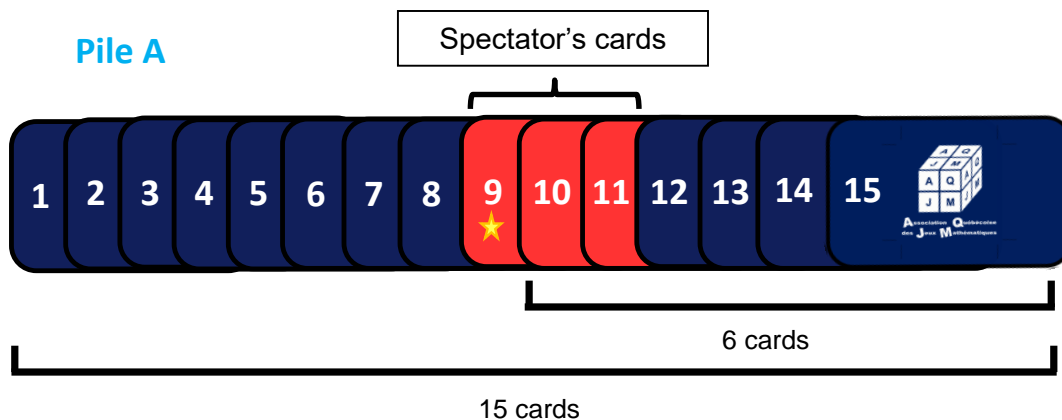
Rest of the deck:

The magician places the 6 cards, one after the other, back on the deck (notice that the spectator's 2nd and 3rd cards will also be placed back on the deck). He takes the following card and sets it aside. This card is the first card chosen by the spectator, which was in the 9th position in the deck.

| The 6 cards put back on the deck | | | | 8 remaining cards | | | |
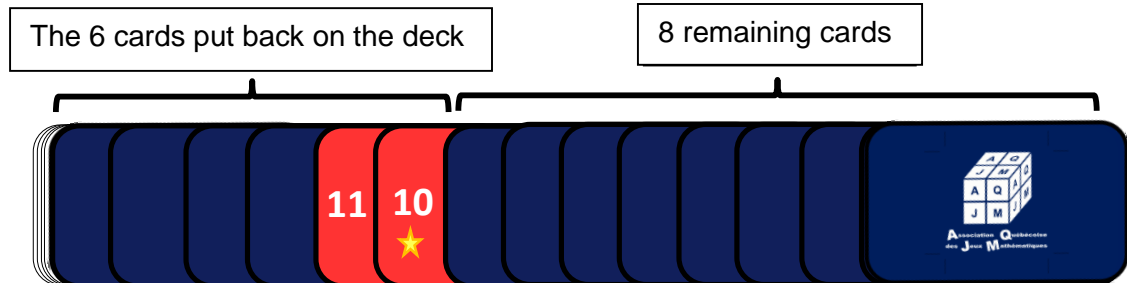
New positions: 14 13 12 11 10 9 8 7 6 5 4 3 2 1

Card set aside: 9
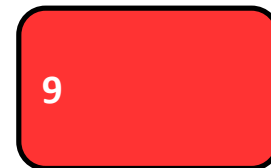
Since the magician set a card aside, the spectator's second card (which was in the 10th position) is now in the 9th position. In the same way, the spectator's third card (which was in the 11th position) is now in the 10th position. So, the magician will be able to set the spectator's second card aside from the deck, because it is in the 9th position, in the same way that was previously explained.

Finally, it is the turn of the spectator's third card (the one that was in the 11th position in the beginning) to get in the 9th position in the deck. The magician can then repeat the same process to set this card aside.

Why do all the numbers from 10 to 19 work?

First of all, since the card we want to take from the deck is always in the 9th position and the number chosen by the spectator is always between 10 and 19, this card will always be placed in **pile A.** Indeed, since this pile counts more than 9 cards (between 10 and 19), the 9th card will necessarily be in this pile.

So, there are always more cards being placed than the desired number of cards in **pile A**. The magician then has to place cards back on the deck to reach the 9th position. How many must he place back?

The following table represents, for each value that the spectator could choose, the number of cards to put back on the deck and the sum of the two digits forming that number.

Spectator's number	Number of cards to put back on the deck	Sum of the two digits forming the spectator's number
10	10 − 9 = **1**	1 + 0 = **1**
11	11 − 9 = **2**	1 + 1 = **2**
12	12 − 9 = **3**	1 + 2 = **3**
13	13 − 9 = **4**	1 + 3 = **4**
14	14 − 9 = **5**	1 + 4 = **5**
15	15 − 9 = **6**	1 + 5 = **6**
16	16 − 9 = **7**	1 + 6 = **7**
17	17 − 9 = **8**	1 + 7 = **8**
18	18 − 9 = **9**	1 + 8 = **9**
19	19 − 9 = **10**	1 + 9 = **10**

In every case, we notice that the values of the last 2 columns are equal.

This can be explained by the fact that the gap (the number of leaps) between 9 and a number from 10 to 19 will always correspond to the units' digit + 1. Since the digit in the tens position is always 1, we simply have to add the digit in the units position to the one in the tens.

This makes the trick more magical for the spectator and it also is a good way to not make a mistake.