



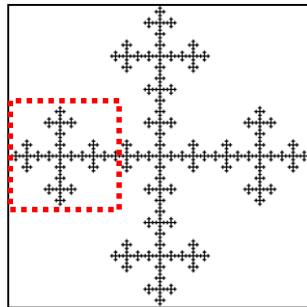
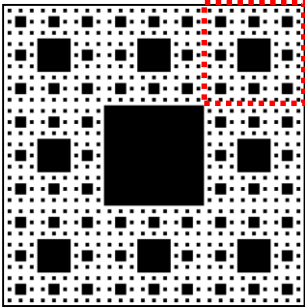
ACTIVITY - FRACTALS -



Step 1: Introduction

Fractals are particular images: they are made of a pattern that repeats itself when enlarging the image. We call this property self-similarity.

Here are two examples:



In both examples, we notice that the pattern framed by red dotted lines is the same as the whole image's pattern.

There are fractals in nature, for example in some plants:



Are you able to find the shapes that repeat themselves in the images?

Step 2: Making a Sierpinski triangle

The Sierpinski triangle is a well-known fractal.

Here is how to draw one yourself!

First, you have to start with an equiangular triangle.

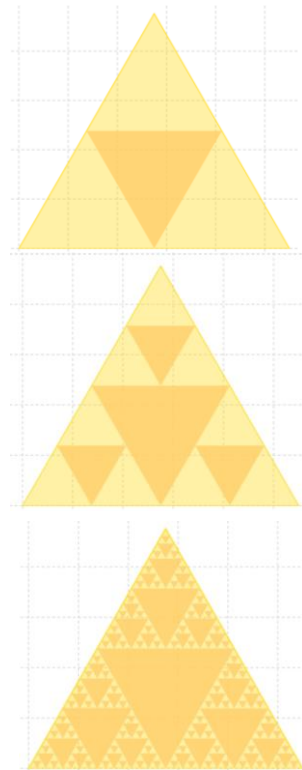


Then, find the midpoint of each side of the triangle and link these three points together. This will create 4 new triangles: the middle one points down and the other three ones point up. Colour the triangle in the middle.

With each of the other three triangles (the ones that point up), you repeat the same procedure you did with the big triangle: link together the midpoints of the three sides to form a triangle that points down and that you will colour.

Keep doing this until you are not able to draw triangles small enough anymore. In the real fractal, the procedure is repeated infinitely.

If you have a computer with an Internet connection, you can use the following link for help: . The Geogebra document that opens shows you the first steps to make a Sierpinski triangle. Use the cursor¹ to go from one step to the other.



Step 3: An interesting property of fractals

Now that you know the Sierpinski triangle better, here is a challenge: are you able to calculate the area of the Sierpinski triangle?

Before jumping to conclusions, try to do the calculation.

You are stuck or want to check your answer? Here is a solution!

One way to calculate the area of the Sierpinski triangle is to notice that, at each step, the area of the new triangle “with holes” corresponds to the $\frac{3}{4}$ of the area of the previous step’s triangle.

So, we can say that at the n step, the triangle’s area is $(\frac{3}{4})^n$ * of the initial area.

With this formula, what happens if n is very big?

Indeed, the bigger n is, the smaller $(\frac{3}{4})^n$ is. And if n is infinite? Then the result is zero.

That’s right! It is the answer we were looking for: the Sierpinski triangle’s area is 0 square units.

Fractals are such particular figures that sometimes they have a null area or an infinite perimeter!

¹ The cursor is at the bottom of the Geogebra page. It is a dot that we can move along a line. By moving this dot, we change the value of “a” and it makes the construction evolve.

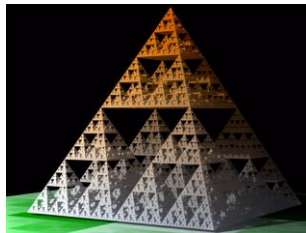
Step 4: Integration

We mentioned earlier that fractals are present in nature. Here is another example: it is a shell with particular patterns that resemble the Sierpinski triangle.



Credit: Simon's specimen Shells Ltd.

So far, we only talked about “two-dimensional” fractals². “Three-dimensional” fractals also exist. An example is the Sierpinski tetrahedron:



² In reality, we cannot really mention two-dimensions and three-dimensions when talking about fractals.