ACTIVITY

- FRACTALS -

Educational Goals

- Introduce fractals
- Develop the mathematical culture
- Observe the presence of mathematics in nature
- Have a broader conception of what mathematics, shapes and figures are
- Broach the notion of infinity

Key Feature of the Targeted Competency

- To build and exploit networks of mathematical concepts and processes

Concepts Used

- Figures (triangles)
- Equivalent figures
- Coefficients of proportionality (comparing areas)
- Fractions
- Power

Materials

- Images of fractals and of a Cymbiola innexa (see appendixes)
- "Sierpinski Triangle" Geogebra document (available on the Internet via this link: https://ggbm.at/dbjrkyfz)
- Sheets of paper
- Pencils
- Ruler
- Protractor

Targeted Academic Level
Grades 9 to 11

Mathematical Fields Concerned

Suggested Teaching Formulas

Time Required
Approximately 25 to 35 minutes

SUGGESTED PROCESS

Step 1: Introduction (10 minutes)

A fractal is a figure made of a pattern that infinitely repeats itself when enlarging the image. We call this property self-similarity. Images representing examples of fractals are provided in the appendix of this sheet.

Show the students that fractals are present in nature by using the images suggested. To guide their observations, pay close attention to the shapes that repeat themselves in the images.

Step 2: Making a Sierpinski triangle (10 to 20 minutes)

The Sierpinski triangle is a well-known fractal. To build it, start with an equiangular triangle. Find the midpoint of each side of the triangle and link these three points together to form 4 new triangles that divide the first one. The middle triangle will point down while the other three will point up. We consider that we are removing the middle triangle (on the representations, we are colouring it). For the other three, repeat the same procedure used for the big triangle, that is linking together the midpoints of the three sides to form a triangle that points down, and then "remove" it. We keep doing this until we are not able to draw triangles small enough anymore.

In the real fractal, the procedure is repeated infinitely. The "Sierpinski Triangle" Geogebra document shows you the first steps. Use the cursor[1] to go from one step to the other.

Grades 10 and 11 (10 minutes)
To bring the reflection further, ask the students to try to calculate the area of the Sierpinski triangle.

One way to do it is to notice that, at each step, the area of the new triangle "with holes" corresponds to the ¾ of the area of the previous step's triangle. So, we can say that at the *n* step, the triangle's area is $(\frac{3}{4})^n$ * of the initial area. It is easy for the students to see that the bigger *n* is, the smaller $(\frac{3}{4})^n$ is. We broaden this comprehension until we realize that if *n* is infinite, the result is zero. It is indeed the expected result: the Sierpinski triangle's area is 0 square units. Fractals are such particular figures that sometimes they have a null area or an infinite perimeter.

[1] The cursor is at the bottom of the Geogebra page. It is a dot that we can move along a line. By moving this dot, we change the value of "a" and it makes the construction evolve.

Step 3: Integration (5 minutes)

We mentioned earlier that fractals were surprisingly present in nature. An example of it is a mollusc which has a shell with particular patterns that remind the patterns of the Sierpinski triangle. That mollusc is a Cymbiola innexa (see the image in the appendix).

So far, we only talked about "two-dimensional" fractals.[2] "Three-dimensional" fractals also exist. An example that is provided in the appendix is the Sierpinski tetrahedron (built in a very similar way than the Sierpinski triangle). Let's mention that the volume of the Sierpinski tetrahedron is null but that it is not the case for its surface.

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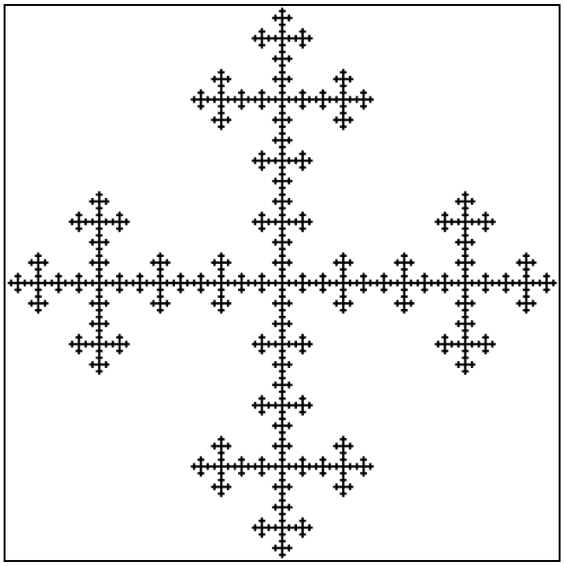
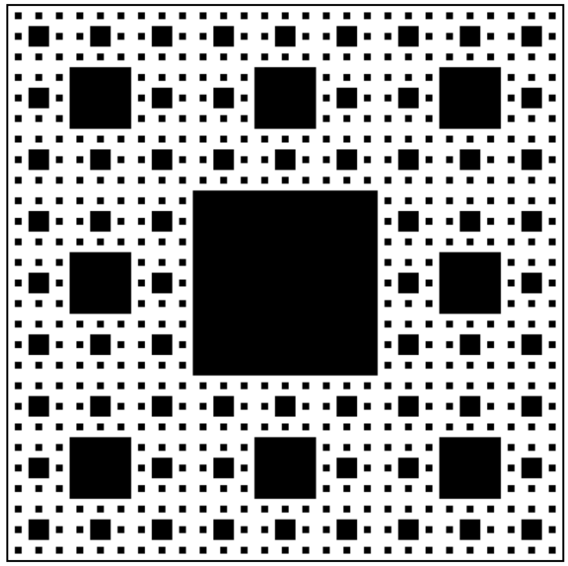
[2] In reality, fractals cannot be expressed in integer dimensions. However, here, we commonly use the expression "two dimensional" to refer to what lies on a plane and the expression "three dimensional" for what lies in a space.

APPENDIXES

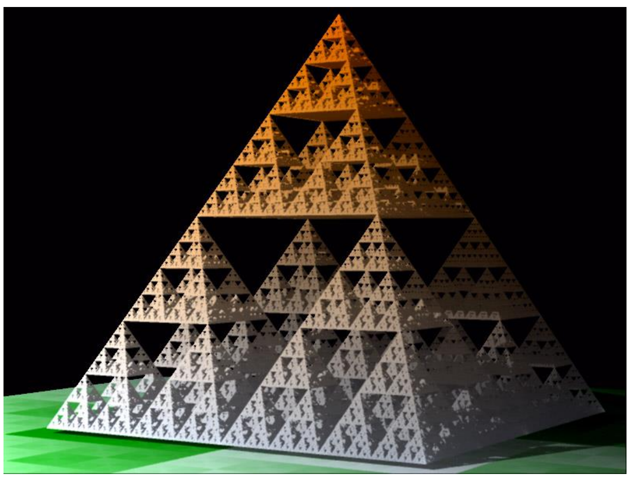
Theoretical Fractals





Fractals in Nature

“3D” Fractals (Sierpinski Tetrahedron)



The Cymbiola innexa

Credit: Simon’s specimen Shells ltd.