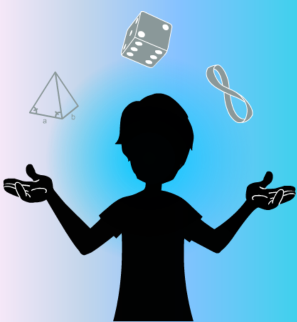


AMAZINGMATHS

Materials:

- A box of multibase blocks or small Lego pieces
- Activity Sheet
- Pencil
- Paper

ACTIVITY

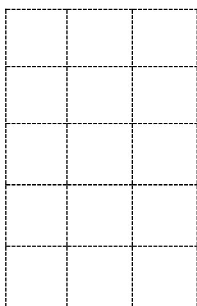
- RECTANGLES, SQUARES -

Instructions

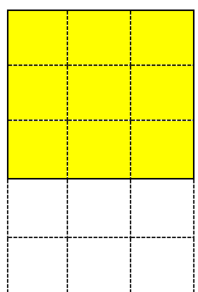
The example's problem:

A rectangular sheet of paper measures 3 by 5. We cut the sheet into two pieces, in a straight line, to get the biggest square possible. We throw away this square and we cut the remaining piece in the same way. We continue cutting that way as long as it is possible. What is the measurement of the smallest square obtained?
Note: all the rectangles and the squares must have whole dimensions (no fractions).

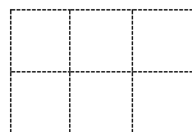
Initial Grid



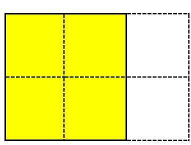
First Cut



Remainder of the first cut

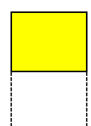


Second Cut

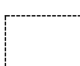


Remainder

Last Cut



Final Remainder:



Dimensions : 1 unit per 1 unit

Solution :

The length of the smallest square's side is always equal to the greatest common divisor (GCD) of the two numbers.

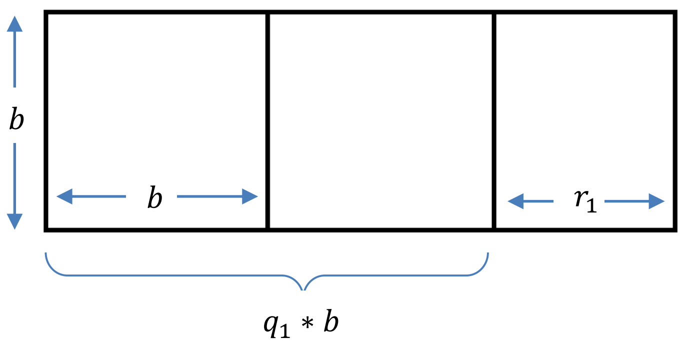
In the example, the GCD of 3 and 5 is 1, because they are prime numbers. The greatest common divisor is then 1. For the 20 by 10 grid: 10 is a divisor of 20. That is why we get two 10 by 10 squares. For the 24 by 15 grid: the smallest square we get has a side of 3, because it is the greatest common divisor of these two numbers.

Detailed Explanations:

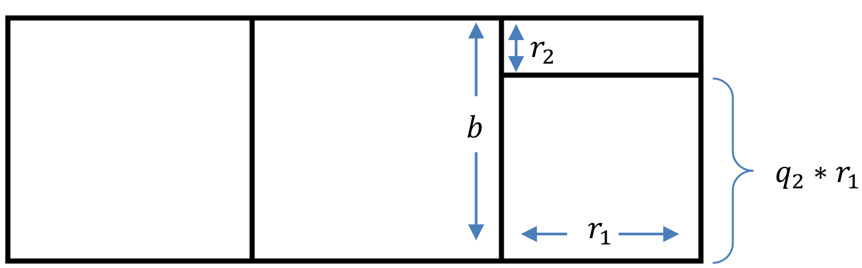
A direct way to see that it is the greatest common divisor (GCD) of the initial measurements is to notice that the last square obtained is the biggest square that could be used to pave the starting rectangle. This allows to visualize that the measurement of the square's side divides the length's and the width's measurements of the initial rectangle at the same time.

Another way to see it is to note that the manipulations done correspond to the Euclidean algorithm to find the GCD of two numbers.

For the first step, the biggest square possible always has as his side's measurement the length of the initial rectangle's smallest side. If the rectangle's longest side is at least twice as long as the smallest one, we will find the same square during the second cut. In fact, if a corresponds to the measurement of the rectangle's longest side and b to the smallest side's measurement, we will be able to reproduce this square as many times as b is contained in a. If we write $a = q_1 * b + r_1$, where q_1 is a certain positive integer and r_1 is the remainder (strictly positive and always smaller than b), we can say that we will find the square with side b q_1 times during our manipulations.



After we have removed all the squares with side b, we are left with a rectangle which dimensions are b on r_1 (because we have removed from the side with the length a the measurement b, q_1 times). We repeat the same process with this new rectangle. We know that the smallest side is the one that measures r_1, and that it fits a certain number of times in b. Let's write $b = q_2 * r_1 + r_2$. So, we will cut q_2 squares with a side of r_1, and the rectangle left will have for dimensions r_1 and r_2.



We continue this way until we obtain a remainder of 0. This means that the squares formed with the previous remainder completely cover the last rectangle. The previous remainder thus corresponds to the GCD of the measurements of the initial rectangle's sides.