

Materials:

- Video of the puzzle
- Sheets of paper
- Pencils

PUZZLING CARTOON

-IMPERFEOT WATERING-

The puzzle

Mr. Blaise has a nice grassy piece of land shaped in a square. To minimize the maintenance, he installs a rotating jet. It waters a circular zone on the lawn. Since it is forbidden to sprinkle the neighbours' property, Mr. Blaise places the jet in the centre of his lawn, so the circle is tangent to the square, as shown on this drawing.



What proportion of the lawn will not be watered?







PUZZLE SOLUTION



The answer:

The exact proportion is $1 - \frac{\pi}{4}$, so approximately 0,2146 or 21,5 %.

The solution:

The circle has a radius of *r* units. Since it is tangent to the square, the side of the square then measures 2^{r} units.

The square's area is given by:

$$A_{square} = (2r)^2 = 4r^2.$$

The circle's area is given by:

$$A_{circle} = \pi r^2$$
.

The area that is not watered is given by:

$$A_{remaining} = A_{square} - A_{circle} = 4r^2 - \pi r^2 = (4 - \pi)r^2.$$

The proportion of the lawn that is not watered is given by:

$$\frac{A_{remaining}}{A_{square}} = \frac{(4-\pi)r^2}{4r^2} = \frac{(4-\pi)}{4} = 1 - \frac{\pi}{4}.$$

Note: To solve this problem without a calculator, we can approximate the value of π by 3,14. In which case, we find exactly 21,5% as the answer.

We obtain

$$\frac{\pi}{4} \approx \frac{314}{400}.$$

So we find a ratio of

$$1 - \frac{314}{400} = \frac{86}{400} = \frac{43}{200} = 21,5\%.$$