PUZZLING CARTOON

- JET LAG -

AMAZINGMATHS

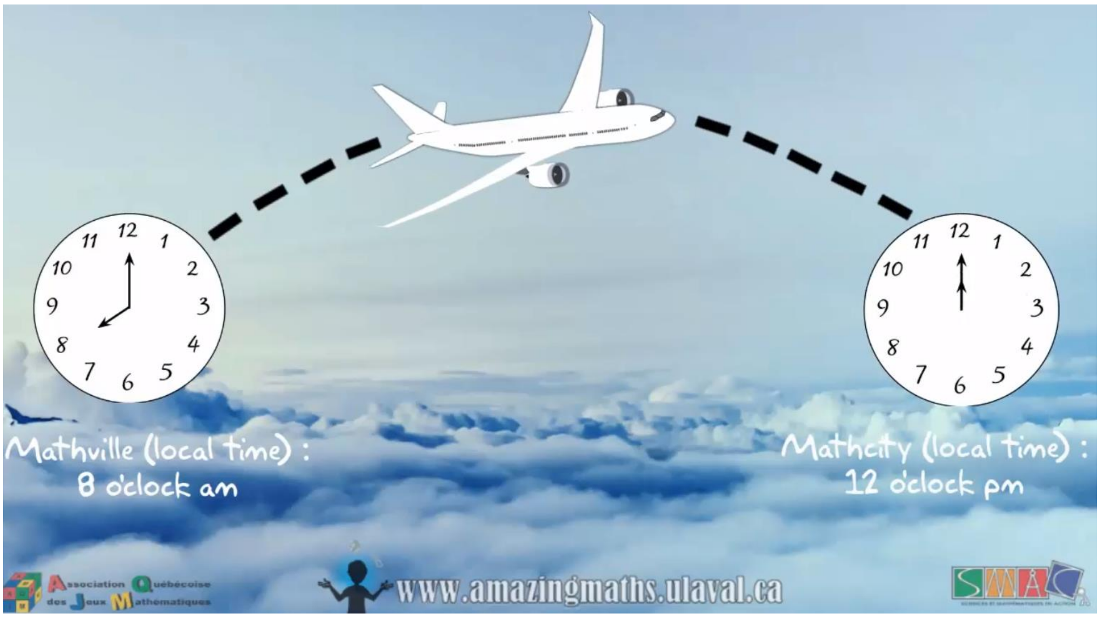
Materials:

- Video of the puzzle
- Sheets of paper
- Pencils

The puzzle

A plane leaves Mathville at 8 a.m. (local time); it lands in MathCity at noon. However, for the trip back, the plane leaves at 2 p.m. (MathCity's local time) and lands in Mathville at 8 p.m.

The duration of both trips is the same, but the two cities are not in the same time zone.



When it is noon in MathCity, what time is it in Mathville?

PUZZLE SOLUTION

The possible answers:

Because the time difference can be of 1 hour, 11 hours or 13 hours, when it is 12 p.m. in MathCity, it is 1 p.m. or 1 a.m in Mathville (with the possibility that it could be 1 a.m. the same morning or 1 a.m. the next morning).

Two possible solutions:

Solution 1:

Let’s imagine we are passengers on the plane to MathCity. We leave Mathville at 8 a.m., local time, and we land in MathCity at 12 p.m., local time. We then leave MathCity at 2 p.m. So, we spent 2 hours in MathCity. The second trip is the same length as the first and it is 8 p.m. when we land in Mathville. Since 12 hours went by in real time between the departure from Mathville and the arrival, and that we spent 2 hours in MathCity, we therefore spent a total of 10 hours on the plane. Each flight is 5 hours long.

Since we travelled for 5 hours to get from Mathville to MathCity, and that the time moved forward only 4 hours, we know that there is a 1-hour time difference between the two cities. MathCity has one hour *less* than Mathville. If it is noon in MathCity, it is 1 p.m. in Mathville.

We must also consider the case in which we come back to Mathville at 8 p.m. *the next day*. There will then be 36 hours (12 + 24) of travelling, including 2 hours spent in MathCity. So, we will have spent 34 hours on the plane, that is 17 hours of flying to go and 17 hours to come back. When we travel from Mathville to MathCity, we spend 17 hours on the plane, while time, once again, moves forward only 4 hours. Thus, there will be a 13-hour (17 – 4) time difference between Mathville and MathCity. MathCity has thirteen hours *less* than Mathville. If it is noon in MathCity, it is 1 a.m. in Mathville, the next morning.

Another case to consider is the following: we leave Mathville at 8 a.m., we land in MathCity at noon *the next day*, then we leave MathCity at 2 p.m. the same day and land in Mathville at 8 p.m., still the same day. The real total duration is again 36 hours, and each flight is 17 hours long. However, during the flight from Mathville to MathCity, while 17 hours go by in real time, the time moves forward 28 hours (24 + 4). This means there is a time difference of 11 hours. This time, MathCity has 11 hours *more* than Mathville. If it is noon in MathCity, it is 1 a.m. in Mathville, the same morning.

If we wanted to keep going this way and consider that the arrival is at 8 p.m. *two* days later, the travelling time would be of 50 hours, and we would then find that there is a 25-hour time difference between Mathville and MathCity, which is impossible, because there are only 24 time zones on the Earth. This means the maximum time difference between two cities is 23 hours. The only three possibilities are then a time difference of 1 hour, 11 hours or 13 hours.

So, if it is noon in MathCity, it is either 1 p.m. or 1 a.m. (the same morning or the next) in Mathville.

Solution 2:
The following solution is more algebraic.

x being the time difference (either positive or negative), the real duration of the flight to go is $4+x$ and the one for the flight back is $6-x$. These two quantities must be equal, which leads to $x=1$. Since we obtain a positive value for x, this means the flight lasted longer than what the time changed to. This means that when we travel from Mathville to MathCity, we go back one hour. If it is noon in MathCity, then it is 1 p.m. in Mathville.

We need to consider two other cases.

First, if the plane that leaves at 8 a.m. from Mathville lands at 12 p.m. in MathCity, then leaves again at 2 p.m. and lands in Mathville at 8 p.m. *the next day*, then the expressions representing the durations of the flights are respectively $4 + x$ and $30 – x$, which gives us $x = 13$. The sign is still positive so, we conclude that, when we travel from Mathville to MathCity, we have to go back 13 hours. Therefore, if it is noon in MathCity, it is 1 a.m. the next morning in Mathville.

Next, if the plane leaving Mathville at 8 a.m. lands in MathCity at 12 p.m. the next day, then leaves again at 2 p.m. and lands in Mathville at 8 p.m. the same day, then the expressions representing the durations of the flights are respectively $28 + x$ and $6 – x$, which gives us $x = -11$. The negative sign indicates that the real duration of the flight is smaller than the difference between the hours displayed on the clocks. This means that when we travel from Mathville to MathCity, we must move forward 11 hours. Therefore, when it is noon in MathCity, it is 1 a.m., the same morning, in Mathville.

There are no other relevant cases to consider. Indeed, by considering cases with more than two days apart, we find the same values for x. In context, this means real durations of flight longer than 24 hours, which cannot happen.