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## The puzzle

## Materials :

- Video of the puzzle
- Sheets of paper
- Pencils
- Calendars of the full year (available in the school agenda)

In a certain group of friends, no one has the same birthday. However, we noticed a particularity.
If each person adds the number corresponding to the day of their birthday to the number corresponding to their birthday month, everyone has the same result. This result is strictly greater than 34.


How many people, at the most, can there be in this group of friends?

Source : Championnat international des jeux mathématiques et logiques, AQJM, finale de l'édition 2011-2012.

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The answer:
There are, at the most, 8 people in the group of friends.

## Detailed Explanation

Since the sum of the day and the month must be strictly greater than 34 , we know that no one was born in January, February, March and April (the maximum sums we can find using these months are $32,30,34$ and 34 , respectively).

We also know that two people cannot have their birthday on the same month. Indeed, if two people were born the same month, they would also be born the same day, because the sum of the day and the month must always be the same. However, we know that, in this group of friends, there are never two people born on the same day.

We then have one person per month and 8 months that can correspond to the constraints, so 8 people in the group.

Note: There are two ways to distribute the birthdays of these eight people. A first solution is that the people are born on May $30^{\text {th }}$, June $29^{\text {th }}$, July $28^{\text {th }}$, August $27^{\text {th }}$, September $26^{\text {th }}$, October $25^{\text {th }}$, November $24^{\text {th }}$ and December $23^{\text {rd }}$. The sum is then 35 . The second solution is found when the sum is 36 . The birthdays' dates are then the following: May $31^{\text {st }}$, June $30^{\text {th }}$, July $29^{\text {th }}$, August $28^{\text {th }}$, September $27^{\text {th }}$, October $26^{\text {th }}$, November $25^{\text {th }}$ and December $24^{\text {th }}$. All the other dates of the year result in sums lower or equal to 34 , or greater or equal to 37 . The sums greater or equal to 37 entail groups of friends made of less people. So, a sum of 37 is not doable with the month of May, which would then have to have 32 days, nor with June, which would then have to have 31 days. So, for a sum of 37 , the group of friends will be reduced to 6 people. We can check the same way that, for a sum of 38 , there will also be 6 people. A sum of 39 will generate a group of 5 friends, the sums 40 and 41 will generate groups of 3 friends. Finally, the sums 42 and 43 entail that there is no one in the group; 43 is the maximum sum that we can obtain.

