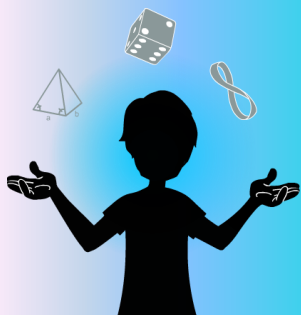


PUZZLING CARTOON

- THE HEN HOUSE -



AMAZINGMATHS

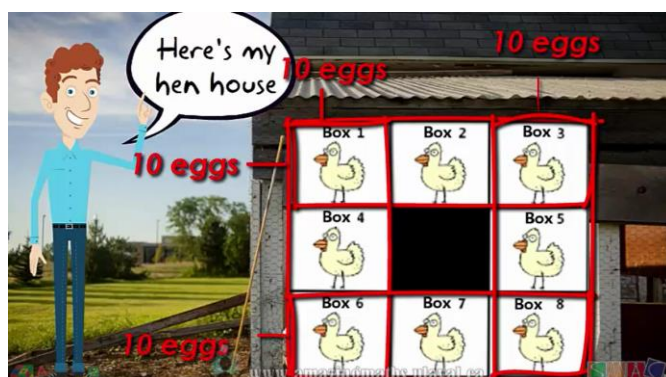
Materials :

- Video of the puzzle
- Sheets of paper
- Pencils

The puzzle

John, our farmer friend, has 8 hens. Each one of them lays eggs in a box of the henhouse.

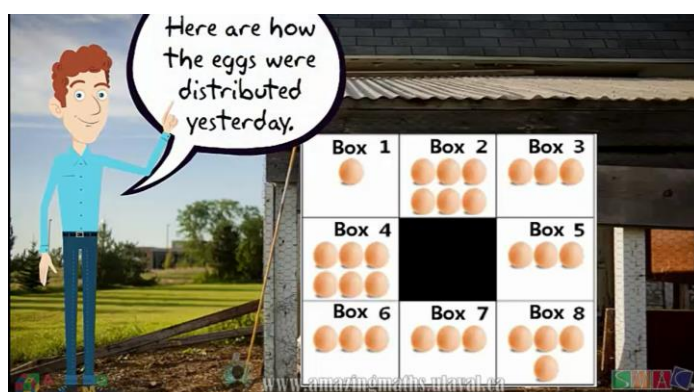
Here is John's henhouse:



It is important to note that each hen occupies one box. These boxes are organized in a way so the farmer always counts 10 eggs in each row and in each column of the henhouse.

For example, the hens laid 29 eggs in total yesterday.

Here are how the eggs were arranged:



John would like to know the minimal number of eggs the hens must lay so each box contains at least one egg and so there is exactly 10 eggs in each row.

Can you find the minimal number of eggs the hens must lay in this case?



PUZZLE SOLUTION





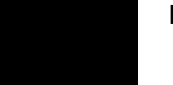






The answer:

The hens must minimally lay **22 eggs** so there is at least one egg per box and so there is 10 eggs in each row and column of the henhouse.

Possible solution:

We should begin by making sure we respect the first constraint of placing one egg per box.

Box 1 	Box 2 	Box 3 
Box 4 		Box 5 
Box 6 	Box 7 	Box 8 




Then, to respect the second constraint, we have to place 10 eggs in each row and each column, so the total number of eggs remains minimal.

We should question ourselves on the impact of adding an egg to a specific box.

Placing an egg in the central box of a row (boxes 2 and 7) means **only** the number of eggs of that row increases. The same thing goes for the central box of a column (boxes 4 and 5).

We notice that the henhouse's boxes that are in the four corners are at the intersection of a row and a column. So, placing the eggs in one of these boxes in the four corners of the henhouse (boxes 1, 3, 6 and 8) means the number of eggs contained in a row **and** a column increases simultaneously. This allows us to use less eggs in total.

So, we want to have only one egg in the central boxes. There are then 9 eggs in total to be placed in the two boxes left of a same row or a same column, and **they can be distributed as desired**.

Case 1 	} Example of a possible distribution of the 10 eggs
Case 4 	
Case 6 	

However, since the corner linking a row and a column counts in the sum of the eggs of each of them, there should be the same number of eggs in the boxes in the opposite corners of the henhouse.



PUZZLE SOLUTION



Among the possible solutions, here is an example. First, we place an egg in each box. Then, we place 8 in a corner of the henhouse to get to the sum of 10 for a row and a column simultaneously. Finally, we place 8 other eggs in the opposite corner, which allows us to also get to the sum of 10 for the remaining row and column.

Box 1 8 eggs	Box 2 1 egg	Box 3 1 egg
Box 4 1 egg		Box 5 1 egg
Box 6 1 egg	Box 7 1 egg	Box 8 8 eggs

Total number of eggs:
 $8 + 1 + 1 + 1 + 1 + 1 + 1 + 8 = 22$ eggs.

The final answer is then 22 eggs. There are other ways to solve the puzzle, but we always end up with this minimal number.

Here are other possible arrangements. The number of eggs contained in the four corners of the henhouse may be inverted in each of them, as long as the boxes in the opposite corners contain the same number of eggs.

Box 1 1 egg	Box 2 1 egg	Box 3 8 eggs
Box 4 1 egg		Box 5 1 egg
Box 6 8 eggs	Box 7 1 egg	Box 8 1 egg

Box 1 8 eggs	Box 2 1 egg	Box 3 1 egg
Box 4 1 egg		Box 5 1 egg
Box 6 1 egg	Box 7 1 egg	Box 8 8 eggs

Box 1 1 egg	Box 2 1 egg	Box 3 8 eggs
Box 4 1 egg		Box 5 1 egg
Box 6 8 eggs	Box 7 1 egg	Box 8 1 egg

Box 1 8 eggs	Box 2 1 egg	Box 3 1 egg
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Box 4 1 egg		Box 5 1 egg
Box 6 1 egg	Box 7 1 egg	Box 8 8 eggs