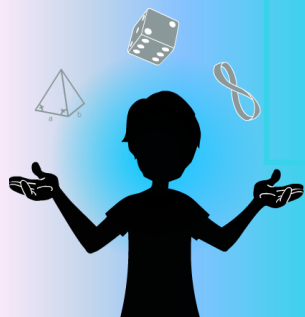


# PUZZLING CARTOON

## - THE LEGEND OF THE BETTING CHIEF -



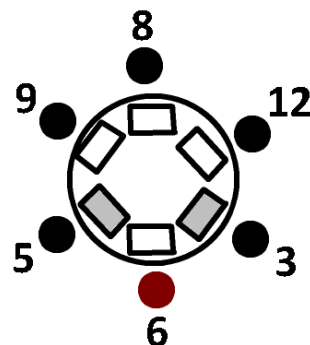
AMAZINGMATHS

### Materials:

- Video of the puzzle
- Sheets of paper
- Pencils

## The puzzle

“Let me tell you the legend of the betting chief... A very long time ago, a group of six adventurers, having just completed a great quest, decided to stop at the nearest village to divide the treasure they had collected. Once arrived, the group’s chief suggested to share the treasure using a little game: they asked a farmer of the village to divide the treasure as he wished into six small chests. Then, each adventurer had to choose a chest, at random, and leave the village... However, at the last moment, the chief, curious, invited the adventurers to sit around a table with their small chest. They did not want to look at their own chest’s content, so each adventurer counted the number of gold pieces of his two neighbours and announced the mean. The first counted a mean of 5 gold pieces, the 2<sup>nd</sup> one of 2 gold pieces, the 3<sup>rd</sup> one of 8 gold pieces, the 4<sup>th</sup> one of 12 gold pieces, the 5<sup>th</sup> one of 3 gold pieces and finally, the chief counted a mean of 6 gold pieces.”



“After that last discussion, the adventurers all left the village. A few hours later, when the adventurers each opened their small chest, the legend says that we heard, across the entire continent, a big scream coming from the group’s chief...”

How many gold pieces were there inside the chief’s chest?



# PUZZLE SOLUTION



## The answer:

There were no gold pieces inside the chief's chest.

## The solution:

### Defining the unknowns

We define the following unknowns:

$u$ : the number of gold pieces in the small chest of the first adventurer.

$v$ : the number of gold pieces in the small chest of the second adventurer.

$w$ : the number of gold pieces in the small chest of the third adventurer.

$x$ : the number of gold pieces in the small chest of the fourth adventurer.

$y$ : the number of gold pieces in the small chest of the fifth adventurer.

$z$ : the number of gold pieces in the small chest of the group's chief.

### Searching for equations

The first adventurer counted a mean of 5 gold pieces among his neighbours. That means that the average number of gold pieces between the ones in the chief's chest and the ones in the second adventurer's chest is 5 gold pieces. We note that:

$$\frac{z + v}{2} = 5.$$

In the same way, we can get another equation with the mean announced by the second adventurer.

$$\frac{u + w}{2} = 9.$$

Continuing this way for the third adventurer, the fourth adventurer, the fifth adventurer and the group's chief, we get the following equations:

$$\frac{v + x}{2} = 8.$$

$$\frac{w + y}{2} = 12.$$

$$\frac{x + z}{2} = 3.$$

$$\frac{u + y}{2} = 6.$$

By multiplying by two each equation we get:

$$\begin{cases} z + v = 10 & (1) \\ u + w = 18 & (2) \\ v + x = 16 & (3) \\ w + y = 24 & (4) \\ x + z = 6 & (5) \\ u + y = 12 & (6) \end{cases}$$

### Solving the equations

With this information, we wish to obtain the number of gold pieces in the small chest of the group's chief, so the value of  $z$ . Let's focus on the two equations that directly use  $z$ : (1) and (5). We can, with them, deduce that  $v = 10 - z$  and  $x = 6 - z$ .

Knowing that  $v + x = 16$ , because of (3), we can now find the value of  $z$ .  
Indeed,

$$\begin{aligned} v + x &= 16 \\ (10 - z) + (6 - z) &= 16 \\ 16 - 2z &= 16 \\ z &= 0. \end{aligned}$$

Thus, we can conclude that the group's chief has no gold pieces in his small chest.

### Note:

Although it may seem natural to take them into account, equations (2), (4) and (6) are not useful to solve the problem. With the equations (1), (3) and (5), we have a system of equations with three equations and three unknowns, which is enough to find the value of this system's unknowns. To see it differently, we can omit what the second adventurer, the fourth adventurer and the group's chief said and we can still solve the puzzle.

